

# Canadas Topographical Survey

# DEPARTMENT OF THE INTERIOR, CANADA

HON. CHARLES STEWART, Minister W. W. CORY, C.M.G., Deputy Minister J. D. CRAIG, Director General of Surveys

F. H. PETERS, Surveyor General and Director of the Topographical Survey

CAI IB46 -29B60

# TOPOGRAPHICAL SURVEY

BULLETIN No. 60



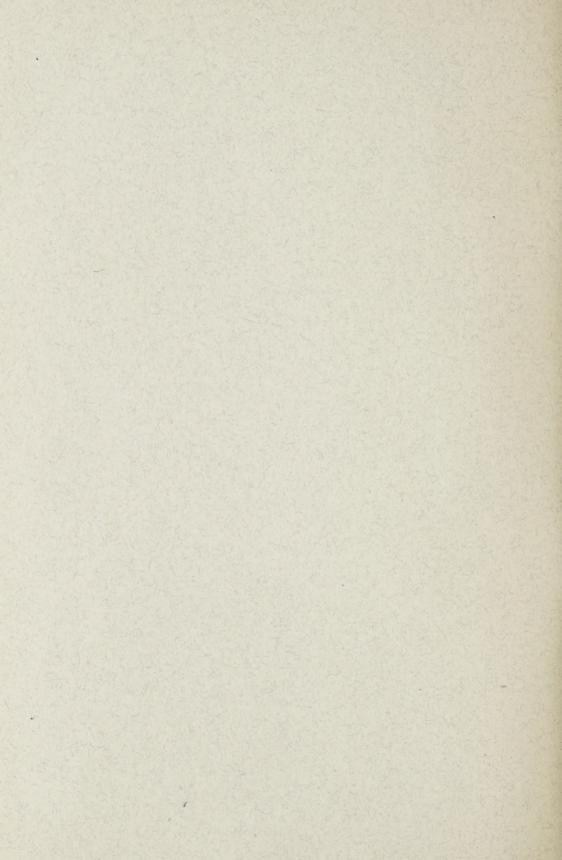
# A STUDY OF THE DOMINION STANDARD YARD AND OTHER STANDARDS OF LENGTH

Issued in co-operation with the Department of Trade and Commerce, Weights and Measures Inspection Service, Standards Branch





OTTAWA F. A. ACLAND PRINTER TO THE KING'S MOST EXCELLENT MAJESTY 1929



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By R. H. FIELD,

Supervisor of the Physical Testing Laboratory

Issued in co-operation with the Department of Trade and Commerce, Weights and Measures Inspection Service, Standards Branch



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#### FOREWORD

By F. H. Peters, Surveyor General and Director of the Topographical Survey, Department of the Interior

Under the provisions of the *Dominion Lands Surveys Act*, the measure of length used in the surveys of Dominion Lands is the Dominion measure of length defined by *The Weights and Measures Act*. Furthermore, every Dominion land surveyor, engaged upon the work covered by his commission, must be in possession of a subsidiary standard which is to be periodically tested under the supervision of the Surveyor General, and stamped as correct by him. The Physical Testing Laboratory of the Topographical Survey includes in its various activities the work of conducting such tests and has the necessary equipment for this purpose.

In this connection the standards of the Physical Testing Laboratory have from time to time been compared with a prototype of the International metre but have never hitherto been compared with the Dominion standard yard,

the legal Dominion standard of length.

This bulletin contains a record of such a comparison conducted during the winter of 1926-1927. At the same time advantage was taken to include in the study a direct comparison between the Dominion standard yard and one of the Parliamentary copies, thus carrying out the provision of *The Weights and Measures Act*. A short examination was also made of a copy of the Imperial standard yard received in Canada some seventy years ago when the prototypes of the Imperial standard yard were distributed throughout the world, and also of the Dominion standard metre.

#### FOREWORD

By E. O. Way, Director of Weights and Measures Inspection Service, Standards Branch, Department of Trade and Commerce

Owing to the fact that the Weights and Measures Branch of the Department of Trade and Commerce is the repository of the Dominion Standards of Weights and Measures, the Surveyor General has been good enough to consider this detail as sufficient justification to extract a foreword for the accompanying publication from me. In these days of specialization and the tremendous expansion of scientific and engineering knowledge, it is becoming increasingly difficult for those engaged in the sciences or practical arts to keep in the van of ceaseless modern progress. In this way, the complicated and intricate developments in modern machinery for weighing and measuring the multiple and diverse commodities of trade and industry to-day has absorbed all the attention of Weights and Measures officials, whilst the Topographical Survey has, for like reasons, found the development of the latest scientific means of comparing and establishing the accuracy of their measures of length one of their first and most important duties. In short, we have here a simple illustration of that natural division of labour which is the inevitable corollary to the advance of modern science.

The physicists of the Physical Testing Laboratory have produced a highly technical and valuable report on the comparison and value of our Dominion Primary Standards of Length. Quite appropriately they have offered a few synoptic remarks upon the early origin of the British Yard and I hope it will not be considered redundant if I presume to extend this side of an interesting subject a little further, for what our predecessors found important has become increasingly so in these modern times of facile international intercourse and

interdependence.

The story of Weights and Measures is largely the story of civilization. The family, the city, the nation, and Empires have all existed and been built upon barter and trade, so that nothing enters so intimately into the life and activities of a people as Weights and Measures. Every step forward in the march of civilization and science has witnessed a corresponding advance in the accuracy and refinement in the implements of commerce, the units and standards of Weights and Measures, at least since the days of the Babylonian, Assyrian, and Egyptian dynasties, for it is generally conceded that the Egyption "cubit" was deduced from a measurement of the earth with extraordinary precision, for the base of the Great Pyramid is 500 cubits long and its perimeter exactly half a meridian mile.

Authorities on the history and antiquity of Weights and Measures are almost all in agreement that the English system of Weights and Measures can be traced back to the Assyrian and Babylonian Empires, by some authori-

ties to as far back as 7,000 B.C.

From the Egyptian cubit came the Egyptian foot, which was one-sixth of the Egyptian meridian cubit of 4 palms or 16 digits, and equal to 12·16 English inches.

This Egyptian foot measure undoubtedly spread all over Europe, no doubt through the trading activities of the early Phoenecians, for we find the Greek, Roman, French, and English foot all closely related to it, as follows:—

Greek foot	 	 	 	 	 	12.1 inches
Roman loot	 	 				11.65 "
french foot.	 	 				12.81 "
English foot.	 	 	 	 	 	12.0 "

4

It is also of interest to note that the Russian unit of length, the Sagene, is exactly equal to 7 English feet, and that the "Hath" in India is equal to 18 inches.

The importance of standards and uniformity in Weights and Measures manifested itself very early. Even as far back as the Saxon kings, royal ordinances are to be found adjuring the people to use "one weight and one measure, whilst reference to the same subject was included in that historical document, Magna Charta (1215). But as Mr. Field says in the accompanying pamphlet, it was not until the reign of Henry VII (1496) and of Elizabeth (1574) that anything approaching scientific attention was given to the construction of standards, both monarchs appointing special commissions for the purpose. So well apparently did these commissions fulfil their task, that the Elizabethan standards remained the legal standards of England until 1824, when one of the most important Weights and Measures Acts of England was passed, which, in addition to establishing a yard made in 1758 as the Imperial Standard yard, introducing the word Imperial for the first time, also established the Standard Avoirdupois pound of 7000 grains, and the Imperial Standard Gallon of 277.274 cubic inches, abolishing the Wine and all other gallons and measures of capacity.

From 1824 on, British standards have been the study of modern metrology, which might be regarded as having its beginning as a modern science with the research work incidental to the introduction of the Metric System in 1790.

Whilst the unit of mass is possibly the most important standard in trade and commerce proper, the unit of length is certainly the most important scientifically and industrially by virtue of the element of permanence and repetitional replacement inseparably associated with its use.

In survey work of course it is the basis of all things, hence the importance that metrology has assigned to the establishment of a standard for the unit of length that shall possess such qualities as those defined by Sir John Herschell in 1863.

"We are driven then, in our choice of a universal standard, to the selection, either of some individual object (if such there be) natural or artificial, imperishable in its nature, unsusceptible of variation by lapse of time or decay, and indestructible by accident or loss; to some ideal or resultant length or magnitude susceptible by its definition of being, as it were, translated into a material expression, and marked out as the result of some process which we are sure will, in all ages and places, reproduce the same identical result. And besides these qualities of invariability, indestructibility, and identical reproducibility, it ought to possess some obvious claim to general acceptance as of common interest to all mankind, an interest from which all national partialities and rivalries should be altogether excluded."

Such properties can only postulate some constant in nature, and of these, Sir John Herschell suggested,

- 1. The Polar axis of the earth,
- 2. The equatorial circumference, as representing material objects of reference, and
  - 3. The velocity of light,
  - 4. The length of a ray of light of some definite refrangibility.
  - 5. The length of the seconds pendulum.

Terrestial measurement had already failed, having been chosen and abandoned in the creation of the Metre, and the seconds pendulum had also been tried and found wanting, but it is interesting to note that Sir John anticipated or hit upon the ultimate indestructible and immutable source of the unit of

length, viz., the wave length of a ray of light, such as was later developed by Professor A. A. Michelson of Chicago University in 1892-3, using the red radiation of cadmium. With the discovery and perfection of this method, material standards will lose their old time importance, as any standard can now be reproduced with reference to this chosen wave length of light, to the utmost precision. The wave length of red cadmium light as determined by Professor Michelson in 1893 is ·64384722μ, whilst later observations by Messrs. Benoit, Fabry and Perot in 1906 gave the wave length a value of ·64384696μ, from which the number of wave length in the metre and yard are given as follows:—

Metre= $1,553,164 \cdot 13$  wave lengths. Yard= $1,420,212 \cdot 04$  wave lengths.

by virtue of which it is claimed that the metre can be defined, in terms of light waves, with an accuracy of 1 part in 10,000,000.

Unfortunately, Weights and Measures proper have not much to do with the scientific refinements of prototype standards; such infinitesimal values as the micron having no concern for the trader or the manufacturer of trade appliances for weighing and measuring. Confederation in 1867 saw the establishment of a Federal Service of Weights and Measures as a sub-branch of the Department of Inland Revenue (merged since 1918 into the Department of National Revenue) and the primary Dominion standards of Canada, then procured and the subject of the comparisons described in the accompanying pamphlet, were placed in the custody of the Minister and direct care of the Commissioner of Inland Revenue, at that time Col. A. Brunel. For a few years, great interest was taken in the standards and in 1877 Dominion Standard Yard "A" was made the subject of comparisons at Ottawa with two American standard yards brought to Ottawa by Mr. J. E. Hilgard of Washington. But as time went on absorption in the collection of revenue and the administration of the Excise services and the affiliated services of Weights and Measures and Gas and Electricity, progressively crowded out interest in the purely metrological side of Weights and Measures work, so that the Weights and Measures Standards Branch gradually ceased to be anything more than the official repository of the Dominion Standards, whilst devoting all its time to the technical, engineering, and administrative supervision of weights and measures, a field in which modern inventive genius is exercised to the full in the development of automatic, time and labour saving devices, functioning with ever increasing efficiency and refinements of accuracy, so that the huge production of to-day can be weighed, checkweighed and distributed with the minimum of cost and the minimum of inaccuracy.

For the preservation of the standards, the Weights and Measures Act has always prescribed that the Departmental standards and Primary standards shall be compared amongst themselves every five and ten years respectively. This work has been done in the past more or less irregularly through the Department calling in a Professor from Toronto University, in the early days, Professor W. J. Loudon, and more recently Professor J. C. McLennan.

In the absence of a Bureau of Standards, the Topographical Survey developed their Physical Testing Laboratory as indispensable to the maintenance of that degree of accuracy in standards of length imperative to survey work. Their need was immediate and obvious, whereas under Weights and Measures, in the absence of any demand for such comparator work, apart from the quinquennial examination of the standards, the creation for such an expensive laboratory has always lacked unequivocal justification. This Branch, however, has always been pleased to co-operate with the Topographical Survey and to recognize their prior interest in this comparator work, and to regard such co-operation as anticipating the co-ordination of the technical services in

the Government under a Bureau of Standards. Thus, the request of the Topographical Survey to compare their survey rules with the Dominion Standard Yard "A" in the custody of this Department, and Parliamentary copy "B", in the custody of the Senate (copy "C" being lost since the House of Commons fire in 1916) met with a cordial response, not only in the interest of the Survey, but as presenting an opportunity of ascertaining the value of the Dominion Primary Standards, after a lapse of over fifty years, in terms of the Imperial Standard Yard, through the intermediacy of the rules in the possession of the Surveyor General, which had recently been compared with the Imperial Yard, as set forth in the pamphlet.

Concluding, it is congratulatory to note that these Confederation standards are in an excellent state of preservation and that in spite of the greatly increased precision in the methods of comparison to which they have been subjected, under the latest scientific methods and apparatus of the Physical Testing Laboratory, they are still remarkably accurate, neither showing an error appreciably greater

than three ten-thousandth parts of an inch.

# CONTENTS

Introduction	
THE IMPERIAL YARD	
The Canadian Yard.  Legal Position.  The Canadian Standard Yards.  Length Comparator.  Later Weights and Measures Acts affecting the Yard.  United States Yard.	1 1 1 1 1
Comparison of the Canadian Legal Yard Standard With the Standards of the Physical Testing Laboratory in 1926-27.  Apparatus Used in the Study of the Standards.  Length Comparator.  Standards of Length of the Physical Testing Laboratory.  Scale of Temperature.  Adjustment of Microscopes.  The Observations.  Method of Support.  Program of Comparisons.  Adjustment of Observations.	10 10 10 10 10 10 10 20 22 22 22 22 23 27
THERMAL DILATATION OF THE DOMINION STANDARD YARD "A".  Dilatation of Baily's Bronze	28 28 30 30
Comparison of the Lengths of the Dominion Standard Yard "A" and the Parliamentary Copy "B"	33
Examination of the Bronze Standard Metre.  The Mètre des Archives.  The International Metre.  Position of the Metre in Canada.  The Canadian Bronze Standard Metre.  Results of the Examination of the Canadian Bronze Standard Metre at the Physical Testing Laboratory.	36 36 36 37 37
EXAMINATION OF THE BRONZE YARD No. 16	39

# INTRODUCTION

The purpose of the present pamphlet is to describe some recent investigations and comparisons made at the Physical Testing Laboratory in connection with the Dominion legal Standards of Length, including the Dominion standdard yard "A", in the custody of the Department of Trade and Commerce, Parliamentary copy "B" in the custody of the Speaker of the Senate, a bronze standard metre and a bronze yard, the latter designated as No. 16, all of which are more particularly described in the following pages. The comparisons were carried out with the utmost care and under the advantages of modern apparatus and technique, and the results may fairly be regarded as establishing reliable values for the length of the Dominion standard yard in terms of the Imperial standard yard, and by inference, of the International metre—the two principal standards of length in use to-day.

As preliminary to our investigations, however, it might not be out of place to refer to the distinction that is drawn between a "unit" and a "standard." Mr. H. W. Bearce, of the Bureau of Standards, Washington, has pointed out the need for avoiding confusion in the use of these terms. He defines them as

follows:-

"A unit, as applied to length measurements, is a nominal distance in space, fixed by definition. It is independent of temperature, pressure, or other physical condition."

"A standard is the physical embodiment of a defined unit. In general, a standard is not independent of temperature, or other physical conditions. It is a true embodiment of the definition it purports to represent, only under definite, standard conditions."

Thus a unit is somewhat analogous to a point or a straight line in mathematics—something that can be discussed, but which it is very hard to define satisfactorily owing to the difficulty of forming a mental picture of it without reference to some material or visible object. The standard is more like the pencil dot or streak which we use to represent the mathematical point or line. We know they have appreciable size or width, yet it is usually possible to make them answer for the ideal concepts within limits. As the limits become more refined, so do the difficulties in making the marks answer for their ideals become greater.

In the case of length, the unit is usually referred to some bar of metal with defining lines ruled on its surface. The accuracy with which such a standard represents the unit will be affected by such causes as temperature, possibility of actual alteration in the length of the bar, width or form of the defining lines, method of support, etc. In proportion to the degree of accuracy required, so does the interference of these latter factors add to the difficulties of the metro-

logist.

#### THE IMPERIAL YARD

Since the Dominion standard yard is derived directly from the Imperial standard yard of Great Britain, a few historical notes on the latter will be of interest as tracing the source and origin of our Dominion standard.

The exact origin of the British yard is uncertain, but it would appear that the length of this unit is for many practical purposes the same as that in use several hundreds of years ago. Various laws have been passed from the time

<sup>\*</sup> Scientific Paper No. 535, "A Fundamental Basis for Measurements of Length".

of Edward I onward establishing standards representing the yard. The earliest authentic standard in existence is said to be a brass yard of the time of Henry VII, now preserved at the Standards Department, Board of Trade, London, England. At the same institution there is also preserved a brass yard made in 1587 by royal command of Queen Elizabeth. The Henry VII yard is stated by the Deputy Warden of the Standards, London, to be 0.037 inch in error when referred to our present standards, while the Queen Elizabeth yard agrees with these standards to within 0.01 inch. Both these old standards are end standards, one yard in length from tip to tip, whereas the present-day standards have fine lines ruled on their surfaces to indicate the length they define.

In 1824 an Act of Parliament was passed which repealed all previous Weights and Measures statutes since the time of Edward I. At the same time, this Act established a standard made in 1758 with reference to the above mentioned Elizabethan yard, and under the instructions of a parliamentary committee, as the Imperial standard yard. In case of loss or destruction, the Act of 1824 ordained that this new Imperial standard yard should be reconstructed by reference to "the pendulum vibrating seconds of mean time in the latitude of London, in a vacuum at the level of the sea, in the proportion of 36 inches

to 39.1393 inches."

Unfortunately, recourse to this latter provision was soon to be made, for barely ten years later the Imperial standard yard was lost in the destruction of the British House of Parliament by fire in 1834. In 1838 a Royal Commission was appointed to report upon the question of constructing new standards, and very early in their deliberations found that the exact length of the seconds pendulum, as deduced from the old standard yard was subject to considerable error, so that they recommended that a new standard be made by reference to existing copies of the old one. It was also recommended that a number of prototypes of the new standard be constructed and distributed, to ensure that a further replacement could easily be made should the necessity arise.

To carry out the recommendations of the Commission a committee was appointed, which numbered in its composition several prominent scientists of that day, such as Sir George Airy, Lord Rosse, Sir W. Herschell, Mr. F. Baily, and the Rev. R. Sheepshanks. The last two gentlemen conducted most of the tests and investigations, and after their death, the work was completed by Sir

George Airy, at that time Astronomer Royal.

The committee investigated various existing yard standards of more or less authenticity, and decided that while a new standard could be made which would, for most practical purposes, be the same length as the one lost, it would be impossible to ensure that its length would represent exactly the old yard unit.

The next consideration was the material of which the new standards should be made, and after careful study of the substances then available, it was ultimately decided to use a bronze of the following composition:—

Copper	 	 	 	 	16 parts
Tin	 	 	 	 	$2\cdot 5$ "
Zinc	 	 	 	 	1 part

This alloy has since been known as Baily's metal, taking its name from the member of the committee who carried out the experiments and research resulting in its adoption.

One serious difficulty the Committee had to face was the question of temperature. At that time temperature scales depended on the thermometers—there was no recognized independent standard. While special thermometers were made for their researches, it cannot be said with certainty that the temperature of 62° F., which the Committee employed as a reference tempera-

ture, could be considered the same as the corresponding figure derived from the gas scale of temperature. This subject is discussed further when dealing with the recent study of the Dominion standards of length (page 20).

The following is the description of the standard yard finally adopted by the Committee and is incorporated in the British Weights and Measures Act,

1878 (41 and 42 Viet. c. 49).

"The Imperial standard for determining the length of the imperial standard yard is a solid square bar, thirty-eight inches long and one square inch in transverse section, the bar being of bronze or gun-metal; near to each end a cylindrical hole is sunk (the distance between the centres of the two holes being thirty-six inches) to the depth of half an inch, at the bottom of this hole is inserted in a smaller hole a gold plug or pin, about one-tenth of an inch in diameter, and upon the surface of this pin there are cut three fine lines at intervals of about onehundredth part of an inch transverse to the axis of the bar, and two lines at nearly the same interval parallel to the axis of the bar; the measure of length of the Imperial standard yard is given by the interval between the middle transversal line at one end and the middle transversal line at the other end, the part of each line which is employed being the point midway between the longitudinal lines, and the said points are in this Act referred to as the centres of the said gold plugs or pins; and such bar is marked 'copper 16 ozs., tin  $2\frac{1}{2}$ , zinc 1. Mr. Baily's metal. No. 1 standard yard at 62.00° Fahrenheit. Cast in Troughton & Simms, London'".

Four similar bars, known as the "Parliamentary Copies," which had been verified in terms of the Imperial standard above defined were deposited at various places in and near London, to serve as reference standards should the legal primary standard be destroyed. One of these Parliamentary copies was immured in the Houses of Parliament, London. A fifth Parliamentary copy was constructed later under the power conferred by Section 5 of the 1878 Act, and is in the possession of the Board of Trade, London, England.

These standards (with the exception of the immured one) are required to be compared between themselves once in 10 years, and with the Imperial

standard once in 20 years.

Some seventy bars, used by the Committee in their investigations, were distributed to various parts of the world, each bar being stamped with the temperature at which it was exactly one yard in length. Of these bars, Canada received No. 16, standard at 61.94° F. No. 16 is a bronze bar, of the same form as the Imperial standard yard and is in the custody of the Director of Weights and Measures, Department of Trade and Commerce, Ottawa.

# THE CANADIAN YARD

**Legal Position.**—The legal standard yard for Canada was intended to represent the same unit as the Imperial standard yard. The Canadian Weights and Measures Act of 1873 reads in part as follows:—

"The 'Imperial Yard' shall be the standard measure of length wherefrom all other measures of length......shall be derived....."

and also:-

"The Minister of Inland Revenue shall cause to be prepared three sets of primary standards of length......and shall cause the same to be duly verified and authenticated in such manner as he shall deem best." "And the Governor, upon being satisfied of the accuracy of the primary standards may, by Order in Council, declare the same to be the legal and only primary standards of length......for Canada, under the name of "The Dominion Standards"....."

The Canadian Standard Yards.—In accordance with these provisions three yard bars were constructed in London, England, by Troughton and Simms, which firm had previously carried out the mechanical work for the Restoration of Standards Committee of 1838. The new standards were constructed of Baily's metal and it was at first intended that they should be made with the X or Tresca cross-section, which was adopted later for the International metre standard and its prototypes. Difficulties were experienced, however, by the makers in constructing the bars to this section, and on the advice of the Deputy Warden of the Standards, London, the Tresca form was abandoned and a square shape adopted, making the new standards both in material and appearance similar to the Imperial standard yard.

A careful verification of the new Canadian standards was carried out by the Deputy Warden of the Standards, Mr. H. W. Chisholm, in 1874. An account of the operations was published at that time by Her Majesty's Stationery Office, London, under the title:—"An Account of the Construction and Verification of Standard Weights and Measures for the Government of Canada."

The three rules are distinguished by the letters A, B and C, engraved on each respectively, in the middle of the upper surface of the bar. They are also engraved, in smaller capitals:—

MR. BAILY'S METAL STANDARD YARD TROUGHTON AND SIMMS

The rules were compared by Mr. Chisholm with standard yard No. 6 of the British Standards Department, which was one of the bronze bars studied by the Restoration of Standards Committee. Its length was found to be identical with that of the Imperial standard yard. A second comparison, made in 1869, showed the two standards still to be identical in length.

Mr. Chisholm in his reports gives the following results. The bars were compared while immersed in water, side by side, and no alteration was made of their relative positions. Each result is the mean of three comparisons, and each comparison is the result of three separate readings of each defining line:—

#### COMPARISON OF STANDARD YARD "A" WITH No. 6

No.	Date	Temp. Mof Ten air b		Mean Results of Comparison	Probable Error of Mean Results
1 2 3	1874 Nov. 2 " 3 " 5 " 6	$57 \cdot 2$ $57 \cdot 2$ $57 \cdot 0$ $57 \cdot 0$	57° · 2 F 57 · 6 82 · 5 36 · 9	inches $ \begin{array}{c} \text{inches} \\ \text{A=No. 6} + 0.00005616 \\ + 0.00001604 \\ - 0.00000437 \\ - 0.00007435 \end{array} $	$\begin{array}{c} \text{inches} \\ \pm 0.00001268 \\ \pm 0.00000880 \\ \pm 0.00000989 \\ \pm 0.00001690 \end{array}$

Owing to the smaller probable error of comparison No. 2, Mr. Chisholm gives the comparison a weight of two. He then computes the length of "A" from comparisons 1 and 2 as:—

<sup>&</sup>quot;A" at 62°F. = Imperial Standard Yard + 0.00002941 inches.

## COMPARISON OF STANDARD YARD "B" WITH No. 6

No.	Date	Temp. of air	Mean Temp. of bars	Mean Results of Comparison Ins.	Probable Error of Mean Results
1 2 3 4	1874 Oct. 29 " 30 " 30 " 31:	59·0 60·0 60·0 61·0	$\begin{array}{c} 58^{\circ} \cdot 4 \text{ F} \\ 58 \cdot 8 \\ 81 \cdot 5 \\ \text{No. } 6 = 34^{\circ} \cdot 0 \\ \text{B} = 35^{\circ} \cdot 0 \end{array}$	B=No. 6—0·00005713 -0·00005103 -0·00017990 -0·00006454	$\begin{array}{c} \text{inches} \\ \pm 0.0001349 \\ \pm 0.00001658 \\ \pm 0.00000603 \\ \pm 0.0000966 \end{array}$

The mean of comparisons 1 and 2 gives

"B" at 62°F. = Imperial Standard Yard - 0.00005419 inches.

Similarly "C" is found to be-

"C" at 62°F. = Imperial Standard Yard + 0.00018668 inches.

Using the value of the coefficient of expansion for Baily's metal determined by Mr. Sheepshanks  $(9.47 \times 10^{-6} \text{ per } 1^{\circ}\text{F.})$  Mr. Chisholm gives:—

"A" at  $61^{\circ}.91F$ . = Imperial Standard Yard. "B" at  $62^{\circ}.16F$ . = Imperial Standard Yard.

"C" at 61°.45F. = Imperial Standard Yard.

Mr. Chisholm concludes from his observations that there is room for doubt as to whether the coefficients of thermal expansion of the Canadian standards are the same as that of No. 6. He recommends that investigations be made as to the amount and possible change in the coefficients with time.

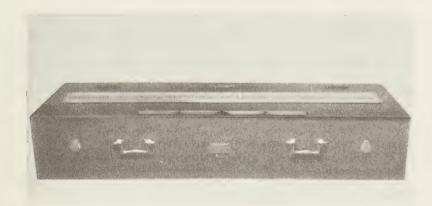


Fig. 1

The safe containing the Dominion standards of length and mass in the custody of the Department of Trade and Commerce.

The three standards were duly received in Ottawa towards the end of 1874, and on December 18 of that year a royal proclamation was issued establishing them as the Dominion standards. "A" was placed in the custody of the Minister of Inland Revenue, "B" with the Speaker of the Senate and "C" in the care of the Speaker of the House of Commons.

The standard "A" was enclosed in a special heavy fire-proof box, which was also made the repository of a standard metre, a standard pound and a standard kilogramme. Fig. 1 illustrates this box, and Fig. 2 pictures the same with the lid open. Standard "A" is the rearmost of the two rules, whilst the standards of mass are to be seen at the right-hand end of the box.



Another view of the safe, Fig. 1, with the lid open. The foremost rule is the bronze standard metre and the other the Dominion standard yard "A". In the pockets to the right are stored a standard pound and a standard kilogramme. The cylindrical objects near the ends of the rules are lids or caps which fit into the end wells for the protection of the defining lines ruled on the gold plugs at the bottom of the wells.

Length Comparator.—Among other standards, secondary standards and apparatus received from England at the same time as the Dominion standards was a comparator for comparing the standards of length among themselves and with other measures. This comparator is illustrated in Fig. 3. The microscope beam is a stone slab, and the microscopes have pierced prisms beneath the objectives for reflecting light received from oil lamps placed behind the apparatus. Four micrometer microscopes are provided, two for observations of yard and similar measures and two, of special construction, for short lengths. Two flat tables are fitted for supporting rules under observation, and in the illustration these carry eight-roller equalizing frames of the type recommended by the Committee of 1838. The rule shown on the nearer roller support is the bronze yard No. 16 already mentioned. Focusing is effected by rods and links operated by the counterweighted handles. The carriage, which runs on three wheels, is traversed by a screw and handwheel.

Later Weights and Measures Acts affecting the Yard.—In 1879 a revising and consolidating Weights and Measures Act was passed (42 Vict. chap. 16). This Act decreed that the bronze bar 'A' should continue to be known as the 'Dominion standard yard', but that the two copies 'B' and 'C' "shall be deemed to be Parliamentary copies of the said Dominion standard" (sect. 5).

In the case of loss and need of replacement, section 6 of this 1879 Act pro-

vided:-

"If at any time either of the Dominion standards of measure and weight is lost, or in any manner destroyed, defaced or otherwise injured, the Department of Inland Revenue may cause the same to be restored by reference to or adoption of any of the Parliamentary copies of that standard, or of such of them as may remain available for that purpose."

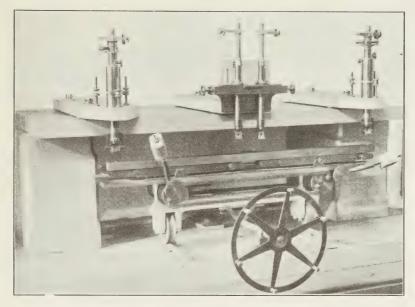


Fig. 3

Old comparator procured in 1874 for verifying standards of length, and now preserved by the Department of Trade and Commerce, Ottawa. Bronze rule No. 16 (a duplicate of the Imperial Standard Yard) is seen supported on the eight roller equalizing frame, in a position for observation.

Section 7 makes a reciprocal provision for the replacement of the Parliamentary copies with reference to the Dominion standard, in case of loss.

Section 32 makes provision for the periodical comparison of the standards as follows:—

"The Department of Inland Revenue shall cause the Parliamentary Copies of the Dominion standards of measure and weight to be compared with each other once in every five years, and once in every ten years with Dominion standards of measure and weight."

"Once, at least, in every five years the Department of Inland Revenue shall cause the Departmental standards, for the time being, to be compared with the Parliamentary copies of the Dominion standards of measure and weight made and approved in pursuance of this Act, and with each other, and to be adjusted or renewed if requisite."

A later Act, passed in 1886, re-affirmed the establishment of "A" as the Dominion standard and of "B" and "C" as the Parliamentary copies and also made the same provision for the inter-comparison of the rules, etc. The same provisions were re-enacted in the Weights and Measures Act, Chap. 52, R.S., 1906.

Of the three Canadian standards, the Dominion standard "A", in the custody of the Department of Trade and Commerce, and the Parliamentary copy "B", in the custody of the Speaker of the Senate, are still in existence, whilst "C", one time in the custody of the Speaker of the Commons, it is now almost certain, was destroyed in the fire of 1916 which burnt the Houses of Parliament at Ottawa.

From the foregoing, it would appear that the Imperial yard, as established by the law of Great Britain, is referred to the length of a specific bronze bar, also, that while it was clearly intended that the unit of length for Canada in the English system should be the Imperial yard, actually the Canadian yard is referred to the length of a certain bronze bar, and may therefore be considered a distinct unit.

United States Yard.—In passing it may be of interest to note that the legal yard in the United States was originally derived from copies of the old British standards, but is now officially defined as \(\frac{3600}{3937}\) of the International metre. This relation makes the United States yard about one part in 363,000 longer than the Imperial yard.

# COMPARISON OF THE CANADIAN LEGAL STANDARD YARD WITH THE STANDARDS OF THE PHYSICAL TESTING LABORATORY IN 1926-7

Since the receipt of the standard yards "A", "B" and "C" in Ottawa in 1874, they have only been compared together once or twice, in air, on the old comparator, Fig. 3, but no check has been made hitherto as to the degree of permanence with which they represent the Imperial yard. It is very difficult to obtain accurate results when conducting comparisons of bronze bars in air, as a slight difference in temperature between the two bars under observation appreciably affects their apparent relative lengths owing to their relatively large thermal expansion (17.6 parts in a million per 1°C.)

In the winter of 1926-7 the Dominion standard yard "A" was verified at the Physical Testing Laboratory, Topographical Survey, Department of the Interior, Ottawa. A comparison was also made between "A" and the sur-

viving Parliamentary copy "B."

## APPARATUS USED IN THE STUDY OF THE STANDARDS

Length Comparator.—The length comparator of the Physical Testing Laboratory, Fig. 4, is housed in a well insulated building,\* electrically heated under thermostatic control. The instrument was constructed mainly in the Instrument Shop of the Laboratory to designs prepared by the technical staff. It consists, like the earlier comparator shown in Fig. 3, of a beam carrying two micrometer microscopes, with a carriage beneath for supporting the rules under observation. The whole apparatus is mounted on massive concrete foundations, isolated from the floor and carried down to rock.

The microscope beam is of cast-iron, the front face being planed so that the microscope holders can be clamped at any desired position. The beam is hollow, and the interior is filled with water which, by increasing the thermal capacity, minimizes the effect of temperature fluctuations and consequent variations in the spacing of the microscopes. Lagging is placed round the beam with the same object. The two piers supporting the microscope beam are independent from those carrying the carriage track, and the beam is held on the top of the piers by cast-iron supports and hardened steel balls arranged on the well-known vee, cone and plane principle. At one end are the cone and plane supports, and at the other, the vee support.

The microscopes magnify about one hundred diameters, and have a working distance of 94 mm. The objectives are by Goerz, with field-illuminating

<sup>\*</sup>A description of the Building is given in Bulletin 44 of the Topographical Survey, Department of the Interior, Ottawa.

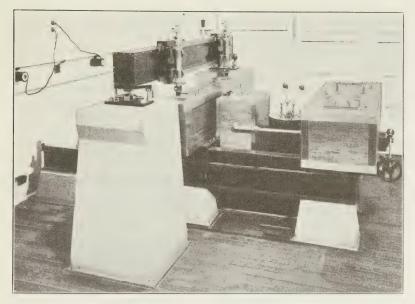


Fig. 4

The length comparator of the Physical Testing Laboratory, Ottawa. To the right is the tank for holding two rules and maintaining them at the same temperature, and to the left, under the microscopes, is the single tank. Between the two tanks may be seen the heating and controlling apparatus used when determining thermal dilatations, etc.

prisms placed between two components. Electric lamps placed behind condenser lenses on the wall are used to illuminate the fields. Before reaching the windows placed before the prisms the light passes through cells fitted with green Wratten filters and ground glass diffusers. The prisms reflect the light vertically downwards. One turn of the micrometer screws, which are fitted with jewelled thrust bearings, is equivalent approximately to 0·1 millimetre at the focus of the objective. The screws are placed eccentrically, to permit the use of long nuts, and have a range of 35 turns. A rack and pinion permits the centering of the eyepiece of each microscope.

Adjustments are provided to the microscope brackets so that the microscopes can be adjusted independently for verticality, while focus, twist, run, etc., are also readily adjustable by positive means. The brackets may be

clamped readily and securely.

The carriage traverses to and fro at right angles to the microscope beam on rails supported from concrete piers. It is traversed by hand, but fine adjustment to the correct position is obtained by a clutch and worm-wheel device acting on the front axle. Two tanks on the carriage contain the supports for the rules under observation. Two rules lying side by side, but supported independently, can be accommodated in the front tank while the rear tank is fitted for one only. Each rule is carried on two rollers, which, with small brackets for holding thermometers close to the rule, are clamped on to a brass girder supported in turn near its ends. Adjustments, operated from outside the tank, are provided for raising or lowering the girders and for moving them longitudinally and transversely, thus permitting rules to be adjusted in the fields of the microscopes for focus, alignment and position. The rule-supporting rollers can also be tilted to adjust rules with speculum finish for horizontality of their graduated surfaces.

The front tank contains an inner trough which is filled with distilled water. In the space between the inner and outer tanks water may be circulated by means of a small motor-driven pump, connected in series with a tank containing electric water heaters wired in a series-parallel arrangement for varying the power from a few watts to about 1,200 watts. The rear tank is single, and in it and in the inner trough of the front tank the distilled water may be thoroughly agitated by means of screw-propellers on vertical shafts operated by hand. As a rule, the temperatures indicated by four thermometers distributed in one of the tanks will agree to 0°·01C.

Two thermometers are supported in the water close to each rule. The thermometers are adjusted to be horizontal and are read by means of a small

vertical microscope through windows in the tank covers.

Standards of Length of the Physical Testing Laboratory.—In the test of "A" a series was made up from four rules, viz. the Dominion standard "A" and Laboratory standard rules 191, 224 and 306. These rules are illustrated in Fig. 5.

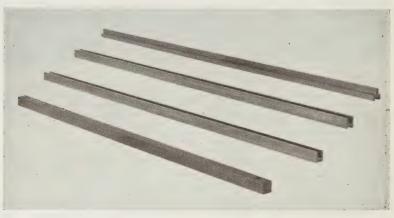


Fig. 5

The four rules intercompared in the course of the verification of the Dominion standard yard "A". Reading from front to rear, (1) is the Dominion standard "A". (2) is one-metre nickel rule No. 306, (3) is one-metre nickel steel rule No. 224 and (4) is one-metre invar rule No. 191.

Rule No. 191 is of invar. 1-04 metres in length. It has an inverted T section and is graduated on one edge of the neutral surface in millimetres and on the other in twentieths of an inch. In this series it was only employed as an intermediate rule, using the interval 2 inches to 38 inches, no assumption being made for the length of this interval. Rule 191 has a low temperature coefficient, the actual equation, as determined by the International Bureau of Weights and Measures, Sèvres, being:—

 $\triangle$  L =  $(2 \cdot 117T - 0 \cdot 00503T^2) \cdot 10^{-6}$ .

Where: △ L=The increase in unit length for a rise from 0° to T° on the hydrogen scale.

Rule No. 224 is of nickel-steel containing 42 per cent nickel. It is of H section, with the graduations ruled along the centre of the exposed neutral surface. The total length of the rule is about 1.03 metres, the height 23.4 mm. and the width 23.4 mm., and it is graduated throughout in millimetres with

an auxiliary millimetre interval subdivided to 0·1 mm. beyond the 0 and 1,000 mm, lines. In addition the rule bears two lines at about 42·9 mm. and 957·1 mm. placed on the rule by the National Physical Laboratory, England, to indicate a nominal distance of one yard. The thermal dilatation of the rule was found at Sèvres to be:

 $\triangle L = (7.544T - 0.00318T^2) 10^{-6}.$ 

the terms having the same meaning as in the case of rule No. 191.

Rule No. 306 is made of pure nickel, but otherwise is similar to rule No. 224. Its thermal dilatation was also determined at Sèvres and is:

 $\triangle L = (12.532T + 0.00566T^2) 10^{-6}$ .

The dilatations of all these three rules have been checked at the Physical Testing Laboratory and the values given above found to hold within the errors of measurement. The three laboratory rules have a speculum finish to their graduated surfaces, which, besides rendering the graduation lines sharp and

even, permits the rules to be readily adjusted under microscopes.

Rules No. 224 and 306 were made by la Société Genevoise d'Instruments de Physique about 1921 and the lengths of the intervals 0 to 1,000 mm. were determined in that year by the International Bureau of Weights and Measures, Sèvres, France, which also determined the thermal equations of the rules, as already stated. In 1924 the rules were again studied at Sèvres and then taken to the National Physical Laboratory, Teddington, England, where the marks defining the yard intervals were engraved and the lengths of these intervals determined. In this determination a series was made up of four rules including one which had been directly compared with the Imperial yard in 1922. The lengths found by the National Physical Laboratory were as follows:

Rule No. 224 = 35.999,142 inches at  $16^{\circ}.667$ C. Rule No. 306 = 36.000,476 inches at  $16^{\circ}.667$ C.

After this test the rules were brought back to Canada personally by the writer and have been compared together from time to time. While a small change of the order of one part in 500,000 took place in their relative lengths after leaving Sèvres in 1921, when they were transmitted by express (no better means being available at that time), there is no evidence of any change of

appreciable amount having taken place since.

The test of the Dominion standard yard was made at the Physical Testing Laboratory during the winter of 1926-7, as already stated. Shortly after the completion of the operations the two rules, Nos. 224 and 306, were carried to England by one of the members of the staff of the Topographical Survey, Department of the Interior, Mr. S. D. Hunt, D.L.S. The two rules were then again verified by the National Physical Laboratory. In this instance the British Parliamentary copy, No. VI, of the Imperial yard was employed in the series. A reproduction of the National Physical Laboratory certificate for No. 224 is given below:—

#### THE NATIONAL PHYSICAL LABORATORY

CERTIFICATE OF EXAMINATION OF LINE STANDARD 224

For:— The Physical Testing Laboratory,
Topographical Survey of Canada,
Ottawa, Canada.

Description:— The bar is of 42% nickel steel with H form cross section. Overall dimensions 1,032 mms. x 23.4 mms. x 23.4 mms. The polished surface in the neutral plane is graduated throughout in millimetres from -1 mm. to 1001 mms., the extreme millimetres being further subdivided into tenths of millimetres. Two addi-

tional lines on the bar at points approximating on the millimetre scale to 42.9 mms. and 957.1 mms. define at 62° Fahr. a nominal yard interval, this being briefly referred to below as the "yard interval".

Marked:-

On top edge "S.I.P. Geneve". On under side, "No. 224, Acier Nickel, 42%; coulée 4118". NPL 52330.

Nature of Test:-To determine the length of the yard interval at 62° Fahr.

The yard interval of bar 224 has been determined by a complete intercomparison of this length and the yard intervals of four other bars, viz.: nickel bar No. 306 belonging to the Physical Testing Laboratory, Ottawa, nickel bar No. 184 belonging to the National Physical Laboratory, 58% nickel bar 1926, and Parliamentary Copy No. VI of the Imperial standard yard, the two last belonging to the Standards Department of the Board of Trade, Westminster.

The comparisons involving P.C. VI have been carried out in the comparator at the Standards Department of the Board of Trade, the remainder being made in the comparator at the National Physical Laboratory.

Each pair of bars has been compared in four different symmetrical positions. The temperature of observations throughout was close to 62° F., the mean temperature being about 61.8° Fahr.

The value of the yard interval of bar 224 has been based on the length of P.C. VI which in 1922 was directly compared with the Imperial standard yard in a closed set of eight bars at the Standards Department of the Board of Trade.

The length of the yard interval of bar No. 224 has been found to be:— 35.999143 inches at 62° Fahr.

This value may be relied on to  $\pm 0.00001$  inch.

The bar was supported throughout at the points marked on its flanks.

For the temperature reductions to 62° Fahr., the following value of the thermal coefficient of expansion of the bar has been employed.

 $L_t = L_0 (1 + 0.000,007,544t - 0.000,000,00318t^2)$ 

in which t is expressed in degrees centigrade, as certified by the Bureau International des Poids et Mesures in June 1921, and as taken from a copy of the certificate supplied to the Laboratory by the Physical Testing Laboratory in 1924. Date: 4th August, 1927

Reference: M.C. NPL 22—89 M. 14911 WHJ J. E. Petavel,
Director.
per SWA

The certificate for rule No. 306 was worded similarly, and the length given was:—

36.000490 inches at 62° Fahr.

It will be noticed that in each case the length agrees with that found in 1924 to within one part in two millions, which is very good evidence of the stability of the two rules. Incidentally the difference between the lengths of the two rules found in 1927 by the National Physical Laboratory agrees almost exactly with the mean difference found by the Physical Testing Laboratory in the course of comparisons made from time to time in the period 1924-7.

Scale of Temperature.—Temperature is akin to length insofar as, to be intelligible, temperature measurements have to be referred to a unit, represented in practice by a working standard. There was no clearly defined unit at the time of the restoration of the British standards of length. Mercury thermometers were in use and it was customary to point the bore at the freezing and boiling points of water subsequently dividing the intervening portion into a number of degrees, each representing the same volume. Unfortunately, glass varies very much in composition and carefully constructed and calibrated thermometers were found to disagree in their indications owing to complexities in the thermal expansion of the glasses of which they were constructed. Then, too, the subject of zero depression and secular changes were not so fully understood as they are to-day. All these things, despite the scientists concerned doing all possible in the light of facts known in their days, introduced some uncertainty in the temperatures they employed.

The unit of temperature now adopted by the International Convention of Weights and Measures is the indication of the gas thermometer, which in the interval 0° to 100° C. is represented to a high order of accuracy by the constant volume hydrogen thermometer. A hydrogen thermometer was set up at the International Bureau of Weights and Measures, Sèvres, France, and a number of carefully made and studied mercury-in-glass thermometers were compared with it and distributed to various countries as working standards for representing the gas scale. More recent research has tended to the reproduction of the gas scale by platinum resistance thermometers verified at three or more points directly or indirectly against a gas thermometer.

With this unit the temperature measurements made by the metrologist become at once intelligible and, what is most important, reproducible. It is customary to assume that the temperature of 62° F. at which the Imperial yard rule is standard is equivalent to 16° 667 C. on the International Temperature

Scale.

At the Physical Testing Laboratory, the temperature scale is represented by a series of mercury thermometers made by Messrs. B. Black and Son of London, who construct many of the precision standards for the National Physical Laboratory. The thermometers are graduated at every 0°.02 C., the length of one degree on the stem being about 25 mm. Before being sent to Canada they were carefully verified against the standards of the National Physical Laboratory, which in turn derives its scale of temperature from some of the glass standards calibrated at Sèvres by means of the hydrogen thermometer.

In the course of the verification of the Dominion standard yard, temperatures were measured by verre dur thermometers made by the late well known French thermometer-maker Tonnelot (whose thermometers are no longer procurable). The Tonnelot thermometers are graduated at every  $0^{\circ} \cdot 1$  C. the length of one degree being approximately 10 mm. With the aid of a small reading microscope they can be readily estimated to  $0^{\circ} \cdot 01$  C. These thermometers are checked from time to time by means of the Laboratory standards, to determine

the corrections to the International Scale.

Adjustment of Microscopes.—Of the various parts of the length comparator probably the micrometer microscopes, with which the actual difference in length between two rules is measured, are the most important. Unless the microscopes are carefully adjusted fictitious results may be obtained. The main points needing attention are (1) screw calibration (2) verticality (3) field illumination (4) run and (5) focus.

Screw Calibration.—The micrometer screws of the two microscopes were calibrated by a method developed at the Bureau of Standards\* which consists essentially in measuring the displacements of the screw under observation by means of a second micrometer microscope. The application of this method showed the screws to be remarkably free from periodic or progressive errors. Moreover, during each comparison the rules were adjusted in such a way that the five or six microscope pointings made on each rule in the course of a single set of observations, were distributed around a complete revolution of the micrometer drum. This system is in use at the National Physical Laboratory, and is aimed at eliminating the effect of accidental as well as periodic errors.

Verticality.—The verticality of the microscope tubes is adjusted with the aid of a sensitive level vial mounted on the top of a rod of the same diameter as the microscopes. Removing a microscope and inserting the rod in its place, the microscope-supporting bracket is adjusted by means of the screws and blocks fitted for the purpose.

<sup>\*</sup> Scientific Paper No 215, "Micrometer Microscopes", by A W. Gray.

Field illumination.—Lamps are placed on the wall at about four feet from the microscopes (see Fig. 4). The lamps are fitted with condensing lenses and the light sources are 6 volt 21 c.p. automobile headlight bulbs. The first adjustment made is to slide the lamps until they are each in a horizontal line passing through the appropriate microscope and perpendicular to the microscope beam. A dish of mercury is then placed beneath the microscope at the position ordinarily occupied by the surface of a rule under observation. Having displaced the cell containing the filter (which is backed by a ground glass diffuser) to one side, the reflecting prism in the microscope is adjusted until the image of the filament (the condensing lenses of the lamps being removed for this purpose) when viewed through a small microscope held above the ocular, is in the centre of the circle defining the exit pupil. When this adjustment has been correctly performed, and the filter-diffuser cell and condenser lens are again placed in the path of the beam of light, the exit pupil should be brightly and uniformly illuminated when viewed through a small auxiliary microscope.

Run.—The value of one turn of each of the micrometer screws was determined by means of standardized millimetre intervals on the invar bar No. 191. These millimetres were calibrated at the International Bureau of Weights and Measures when the rule was originally studied there in 1919. As the tube lengths of the microscopes are readily adjusted and are provided with graduated scales, the values of one turn of the screws were adjusted each to equal 0·1000 mm. During the observations on the rules check tests were made from time to time to ascertain that this value remained constant.

Focus.—To ensure that the rules under observation would be perpendicular to the microscopes, these were adjusted so that their foci were in the same horizontal plane. The operation was readily carried out by filling the tank with water up to the necessary height and adjusting the positions of the microscopes until the minute particles floating on the surface were in focus. The focus of the microscopes is very sharp and definite, so that rules are readily brought to the correct position once the microscopes are adjusted.

#### THE OBSERVATIONS

The Dominion standard yard "A" was compared with the Laboratory rules during the period Nov. 15 to Dec. 14, 1926. When viewed under the microscopes of the comparator the graduation lines on "A" are somewhat irregular and much wider than those on modern rules, e.g. the Laboratory rules. Moreover, the gold plugs are by no means optically flat and have a mottled appearance under the microscopes. It was decided to have three experienced observers each conduct an independent series of comparisons; and, in addition, each observer made his own interpretation of the pointing of the microscope cross-wires to satisfy the condition that the yard is defined by that portion of the graduation marks midway between the longitudinal lines. It should be noted in passing, that in the microscopes the frame operated by the micrometer screws is fitted with two parallel spider webs at the focus of the eyepiece, and pointings are made by moving the screw until the image of the line under observation is exactly in the centre between the two webs.

Method of Support.—Early standards were used while supported on plane surfaces. It was recognized, however, that accidental irregularities in the supporting surfaces introduced errors in the observed lengths of the standards, particularly when the graduations were ruled on the upper surfaces. The Restoration of Standards Committee, besides engraving the defining lines of the new standards on the neutral surface, supported the rules during com-

parisons by floating them in mercury. They provided, however, that in future the rules would be supported on equalizing frames fitted with eight rollers spaced so that the pressure was equal on each roller. The eight roller method of support is rather clumsy, and the International Committee which established the International metre adopted the principle of supporting the rules at two points only. Broch\* computed that for the graduation marks to be at a maximum distance apart, i.e. when slight displacements of the supports will have a minimum effect on the length of the rule, the supports should be at a distance apart equal to 0.56 of the total length. Sir G. Airy† determined that to establish the condition of complete horizontal alignment for both ends of the graduated surface, the distance apart of the two supports should be 0.58 of the total length, which for practical purposes agrees with Broch's spacing. It is the universal custom now to support standards of length at two points, spaced in accordance with one or the other of these figures.

In the observations on the Dominion standard yard "A" at the Physical Testing Laboratory, the bar was supported on two rollers placed at a distance of 21.9 inches apart. Approximate calculations, as well as experimental observations at the Physical Testing Laboratory and at the National Physical Laboratory in England all show that no appreciable alteration in the length of a rule graduated on the neutral surface is introduced when the method of support is changed from the eight roller to the two roller system. Moreover, the two roller system has been employed in recent observations of the Imperial standard yard and its copies in England. Furthermore, the subject has been investigated mathematically by Tresca, who, in a paper presented to the Metric Convention at Paris and reproduced in the Annual Report of the old Inland Revenue Department, Canada, for 1875, shows that the actual shortening of the neutral fibres of a rule supported in this manner is negligibly small.

Adjustments are provided to the roller supports permitting the rules to be tilted slightly about their longitudinal axes. In the case of a rule with speculum finish the tilting is governed by the appearance of the microscope exit pupil, which, when viewed by an auxiliary ocular or other small microscope should be uniformly filled with light. In the case of the yard "A" this adjustment could not be made owing to the lack of flatness of the upper surfaces of the gold plugs. The large number of times the rule was changed during the observations, however, should eliminate the possibility of any error in the final length obtained due to illumination effects.

**Program of Comparisons.**—Each of the three observers compared the four rules, two by two, in every possible combination. Furthermore each pair was compared in each of the eight possible positions they could occupy with respect to each other and to the observer. The order adopted was as follows, calling the rules under observation A B and C D:—

The complete program was, then, for each observer:—

Sets	1	to	8	inclusive,	comparisons	between	rules	224	and	306
66			16		- 66	66	,66	224	66	"A"
66	17	66	24	66	46	66	66	224	66	191
66	25	66	32	66	46	66	66	"A"	66	191
66	33	66	40	66	66	66	66	306	6.6	191
4.6			48	66	66	66	66	306	66	"A"

i.e. the whole comparison comprised 144 sets of observations.

<sup>\*</sup>Traveaux et Memoirs du Bureau International, VII, B62. †Philosophical Transactions of the Royal Society, 1857, pt. III, page 17.

The procedure during a set was as follows: Having brought each rule in turn approximately to the correct position at the focus of the microscopes, the rule was adjusted by tilting the rollers until both exit pupils were as uniformly illuminated as possible. The water in the inner tank was agitated for about thirty seconds and then the thermometers were read. The rule nearest the observer was then brought to correct focus and pointings made successively with the two microscopes. The tank was traversed until the rear rule was brought beneath the microscopes, and a similar operation carried out. This procedure was repeated until six observations had been made on the front rule and five on the rear one. After three double observations had been completed the water was again agitated for about twenty seconds and at the conclusion of the set for about thirty seconds when the thermometers were read a second time. A typical record of a single set is reproduced in Table I.

#### TABLE 1

#### PHYSICAL TESTING LABORATORY

#### Rule Comparison

Communication of	100 2	24 0	
Comparison of	"B" "	T. & S.	Date17-11-26 11,30 A.M.
Observer	s. J. I	л.	Recorded by G. O. W.
Computed by	G. O.	W.	Checked byW.J.L.

#### THERMOMETERS

No	38030	38031	38032	38033	Mean
Reading (1)(2)	16·71 ·72	16·77 ·77	16·73 ·74	16·78 ·79	16·748 ·755
Mean	16.715	16.770	16.735	16.785	16.751
Correction	-0.035 $16.680$	-0.087 $16.683$	-0.054 $16.681$	-0.104 $16.681$	-0.070 $16.681$

#### MICROSCOPES

	Left No. 2		Right	No. 1	L - R	
"A"	20·831 20·565 20·492 19·726 19·502 19·216	$20 \cdot 957$ $20 \cdot 720$ $20 \cdot 272$ $19 \cdot 818$ $19 \cdot 662$	20·554 20·288 20·213 19·447 19·224 18·938	20·880 20·644 20·197 19·747 19·592	+0.277 $+0.277$ $+0.279$ $+0.279$ $+0.278$ $+0.278$	+0.077 $+0.076$ $+0.075$ $+0.071$ $+0.070$
Mean Difference	20·0553 -0·	20·2858 2305	19·7773 -0·	20·2120 4347	+0·2780 +0·204	+0.0738

Micrometer Interval		Microns
Temperature Correction	- 0.13	
Difference corrected to 16° 667 C	+20.29	66

#### TABLE II

Summary of the comparisons of the yard bars employed in the verification of the Dominion Standard Yard "A"

All differences are reduced to 16° · 667 C.

(a)

LENGTH OF RULE No. 306-LENGTH OF RULE No. 224

Set	Observer	: S. J. M.	Observer	: W. J. L.	Observer: R. C. R.		
Det	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	
1	$33 \cdot 90$ $34 \cdot 55$ $34 \cdot 23$ $34 \cdot 28$ $34 \cdot 28$ $34 \cdot 40$ $34 \cdot 65$ $34 \cdot 32$	16° · 861 C 16 · 849 16 · 855 16 · 848 16 · 656 16 · 644 16 · 646	$34 \cdot 09$ $34 \cdot 18$ $34 \cdot 09$ $34 \cdot 18$ $34 \cdot 80$ $34 \cdot 18$ $34 \cdot 80$ $34 \cdot 24$ $34 \cdot 27$ $34 \cdot 30$	16° · 147 C 16 · 144 16 · 165 16 · 174 16 · 218 16 · 225 16 · 247 16 · 250	$34 \cdot 03$ $34 \cdot 09$ $33 \cdot 85$ $33 \cdot 79$ $34 \cdot 05$ $34 \cdot 28$ $33 \cdot 91$ $34 \cdot 38$	16°·089 C 16·068 16·072 16·067 16·104 16·075 16·095 16·090	
Mean	34 · 325	16° ⋅ 750 C	34 · 252	16° · 198 C	34 · 048	16° ⋅ 083 C	

(b)

LENGTH OF RULE "A"-LENGTH OF RULE No. 224

Set	Observer:	S. J. M.	Observer	: W. J. L.	Observer: R. C. R.		
	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	
1	20·50 20·31 19·84 19·69 19·91 20·29 20·26 20·30	16° · 636 C 16 · 618 16 · 624 16 · 666 16 · 679 16 · 681 16 · 680 16 · 700	20 · 36 21 · 09 20 · 91 21 · 81 22 · 18 21 · 64 20 · 88 21 · 17	16° · 365 C 16 · 354 16 · 354 16 · 365 16 · 385 16 · 378 16 · 384 16 · 385	$20 \cdot 89 \\ 20 \cdot 17 \\ 20 \cdot 60 \\ 20 \cdot 29 \\ 21 \cdot 07 \\ 21 \cdot 15 \\ 20 \cdot 96 \\ 20 \cdot 81$	16°·146 C 16·102 16·087 -16·075 16·092 16·104 16·094 16·106	
Mean	20.138	16°·660 C	21 · 255	16°·371 C	20.742	16° · 101 C	

(c)

#### LENGTH OF RULE No. 224-LENGTH OF RULE No. 191

Set	Observer	: S. J. M.	Observer	: W. J. L.	Observer: R. C. R.		
	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	
1	1 · 60 1 · 42 1 · 41 1 · 52 1 · 48 1 · 84 1 · 21 1 · 39	16°·758 C 16·746 16·734 16·732 16·726 16·712 16·711 16·706 16°·728 C	1·65 1·39 1·85 1·71 1·66 1·63 1·98 1·41	16° · 404 C 16 · 395 16 · 398 16 · 392 16 · 392 16 · 418 16 · 418 16 · 420 16° · 405 C	1·81 2·16 1·78 1·56 1·47 1·29 1·76 1·76	16° · 273 C 16 · 160 16 · 145 16 · 145 16 · 139 16 · 136 16 · 170 16 · 173 16° · 168 C	

#### TABLE II-Concluded

(d)

LENGTH OF RULE "A"-LENGTH OF RULE No. 191

	Observer: S. J. M.		Observer	: W. J. L.	Observer: R. C. R.		
, Set	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	
1	$21 \cdot 73 \\ 22 \cdot 02 \\ 22 \cdot 02 \\ 21 \cdot 45 \\ 21 \cdot 38 \\ 20 \cdot 82 \\ 20 \cdot 84 \\ 21 \cdot 09$	16°-688 C 16 -690 16 -694 16 -710 16 -694 16 -676 16 -635 16 -616	$23 \cdot 75 \\ 24 \cdot 01 \\ 24 \cdot 95 \\ 24 \cdot 50 \\ 23 \cdot 13 \\ 24 \cdot 28 \\ 24 \cdot 07 \\ 22 \cdot 84$	16° · 514 C 16 · 512 16 · 520 16 · 497 16 · 528 16 · 532 16 · 537 16 · 538	$\begin{array}{c} 22 \cdot 15 \\ 22 \cdot 84 \\ 22 \cdot 78 \\ 22 \cdot 99 \\ 22 \cdot 69 \\ 22 \cdot 71 \\ 23 \cdot 02 \\ 23 \cdot 40 \end{array}$	16° · 252 C 16 · 245 16 · 242 16 · 241 16 · 245 16 · 266 16 · 292 16 · 294	
fean	21.419	16° ⋅ 675 C	23.941	16°⋅522 C	22.822	16° ⋅ 260 C	

(e)

#### LENGTH OF RULE No. 306-LENGTH OF RULE No. 191

Set	Observer	:: S. J. M.	Observer	·: W. J. L.	Observer: R. C. R.		
	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	
1	35 · 66 35 · 67 35 · 58 36 · 25 35 · 68 35 · 59 35 · 48 35 · 63	16° · 589 C 16 · 584 16 · 589 16 · 588 16 · 742 16 · 740 16 · 735 16 · 732	$36 \cdot 00$ $35 \cdot 76$ $36 \cdot 16$ $36 \cdot 17$ $35 \cdot 88$ $35 \cdot 86$ $36 \cdot 23$ $35 \cdot 80$	16° · 609 C 16 · 594 16 · 600 16 · 594 16 · 597 16 · 611 16 · 621 16 · 625	36·00 35·93 35·95 35·88 35·65 35·43 35·68 35·38	16° · 588 C 16 · 574 16 · 568 16 · 572 16 · 578 16 · 575 16 · 614 16 · 609	
Iean	35.692	16° · 662 C	35.982	16° · 606 C	35.738	16° ⋅ 585 C	

 $\begin{array}{c} \text{(f)} \\ \text{Length of Rule No. 306-Length of Rule "A"} \end{array}$ 

Set	Observe	r: S. J. M.	Observer	: W. J. L.	Observer: R. C. R.		
260	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	
1	$14 \cdot 42 \\ 14 \cdot 19 \\ 13 \cdot 47 \\ 14 \cdot 46 \\ 14 \cdot 67 \\ 14 \cdot 03 \\ 13 \cdot 79 \\ 14 \cdot 83$	16°·716 C 16·700 16·695 16·696 16·679 16·676 16·670 16·672	$12 \cdot 63$ $12 \cdot 75$ $12 \cdot 49$ $12 \cdot 60$ $12 \cdot 62$ $12 \cdot 38$ $12 \cdot 76$ $12 \cdot 72$	16° · 649 C 16 · 656 16 · 648 16 · 659 16 · 655 16 · 669 16 · 630 16 · 626	$\begin{array}{c} 13 \cdot 53 \\ 12 \cdot 76 \\ 12 \cdot 77 \\ 12 \cdot 75 \\ 13 \cdot 08 \\ 13 \cdot 19 \\ 12 \cdot 29 \\ 12 \cdot 58 \end{array}$	16°·786 C 16·770 16·772 16·772 16·789 16·788 16·809 16·806	
Mean	14.232	16° ⋅ 688 C	12.619	16° ⋅ 649 C	12.869	16° ⋅ 786 C	

As the temperature of the comparator room is regulated by a thermostat, the temperature throughout the test was maintained between  $16^\circ$  and  $17^\circ C$ .

All the observations were reduced to 16° 667C. (see Table I).

Table II is a list of the actual results of the 144 comparisons. The effect of the relatively poorer graduation lines on "A" is clearly seen in the variation between the observations of the three observers where that rule is compared.

In the case of the laboratory rules, with their finer and more regular lines, the agreement is much better. The differences between the rules are given in microns (1 micron=0.001 millimetre, or approximately 0.00004 inch). Some idea of the magnitude of the actual differences between the rules and the variations between different observations may be gauged from the fact that one micron is approximately a one millionth part of the length of the rules (actually 1 part in 914400).

Adjustment of Observations.—The results of the comparisons summarized in Table II were adjusted by a tabular method developed at the National Physical Laboratory. A description of the method, which is applicable to any number of quantities compared in a closed series, is given in the Dictionary of Applied Physics, Vol. III, page 239. Fig. 6 shows the actual computation for the observations of S.J.M. (Table II) while in Table III there are listed the observed and compensated differences for each of the three observers.

American Inches				
	306	224	191	"A"
306		$-34 \cdot 32$	-25.69	-14.23
224	+34·31 +34·32 (0·01)		- 1.48	+20.14
191	+35.71  +35.69  (0.02)	+ 1·40 + 1·48 (0·08)		+21.42
"A"	+14.23 +14.23 (0.00)	$     \begin{array}{r}       -20 \cdot 08 \\       -20 \cdot 14 \\       (0 \cdot 06)     \end{array} $	$ \begin{array}{r} -21 \cdot 48 \\ -21 \cdot 42 \\ (0 \cdot 06) \end{array} $	
Sum. Mean.	+84·24 +21·060	$-52 \cdot 98 \\ -13 \cdot 245$	$-58 \cdot 59 \\ -14 \cdot 648$	$+27 \cdot 33 + 6 \cdot 832$

FIG. 6

Adjustment of the results of the intercomparisons of the four rules, 306, 224, 191 and "A" made by S.J.M.

The figure in the centre of each square is the directly observed value of the length of the rule in the column minus the length of the rule in the row. It is readily seen that one fourth the difference between the sums of any two columns is the mean value of the difference between the lengths of the respective rules. This quantity is printed in italics above the directly observed difference, while the figure below in brackets is the residual. All figures are in microns. Table III shows the observed and adjusted values for each of the three observers.

TABLE III

	Observer: S. J. M.			Obs	erver: W.	J. L.	Observer: R. C. R.		
—	Observed directly	Com- puted from Series	Differ- ence	Observed directly	Com- puted from Series	Differ- ence	Observed directly	Computed from Series	Differ- ence
	Microns	Microns	Microns	Microns	Microns	Microns	Microns	Microns	Microns
Rule 306—Rule 224 hule "A"—nule 224. Rule 224—Rule 191. Rule "A"—Rule 191. Rule 306—Rule 191. Rule 306—Rule 4".	$\begin{array}{c} 34 \cdot 32 \\ 20 \cdot 14 \\ 1 \cdot 48 \\ 21 \cdot 42 \\ 35 \cdot 69 \\ 14 \cdot 23 \end{array}$	$34 \cdot 31$ $20 \cdot 08$ $1 \cdot 40$ $21 \cdot 48$ $35 \cdot 71$ $14 \cdot 23$	0·01 0·06 0·08 0·06 0·02 0·00	$34 \cdot 25$ $21 \cdot 26$ $1 \cdot 66$ $23 \cdot 94$ $35 \cdot 98$ $12 \cdot 62$	$34 \cdot 17$ $21 \cdot 60$ $1 \cdot 93$ $23 \cdot 54$ $36 \cdot 10$ $12 \cdot 57$	$\begin{array}{c} 0.08 \\ 0.34 \\ 0.27 \\ 0.40 \\ 0.12 \\ 0.05 \end{array}$	34.05 $20.74$ $1.70$ $22.82$ $35.74$ $12.87$	33 · 94 20 · 94 1 · 79 22 · 74 35 · 73 12 · 99	0.11 $0.20$ $0.09$ $0.08$ $0.01$ $0.12$

The important quantities emerging from these computations are the differences between "A" and 224 and 306 respectively. These are:—

Observer:	S. J.	S. J. M. W. J. L.		R. C. R.		
	Microns	Inches	Microns	Inches	Microns	Inches
Rule "A" – Rule 224		$0.000791 \\ 0.000560$		$0.000850 \\ 0.000495$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Using the lengths for rules 224 and 306 found from the comparison with the Imperial Parliamentary copy, No. VI, the following lengths are derived for "A":—

Observer: S.J.M. W.J.L. R.C.R. Mean Length of "A" from 224 = 35.999934 35.999993 35.999967 35.999965 inches " " " 306 = 35.999930 35.999995 35.999979 35.999968 " giving a final mean of 35.999966 inches at  $16^{\circ}.667$  C.  $(62^{\circ}$  F.)

This indicates an apparent shortening of  $0.000063^*$  inches (one part in 500,000 nearly) since the original standardization. However, in view of the limitations of the accuracy of the original comparisons, and the uncertainty imposed by the form of the graduations on "A", it cannot be said definitely that the length of the Standard "A" has changed since it was originally verified.

Applying the coefficient of expansion found by direct observation on "A" the length of the Dominion standard at 16°·617 C. (61°·91 F.) becomes 35·999934 inches\* in terms of the Imperial standard yard.

# THERMAL DILATATION OF THE DOMINION STANDARD YARD "A"

In length comparator work it is impracticable to make all observations at precisely the same temperature and two standards it is desired to compare may have been originally verified at different temperatures. To reduce observations to some basic temperature, especially when the rules concerned have appreciably different thermal coefficients, demands an accurate knowledge of the thermal dilatations. It is therefore customary, in precise work, to make a direct observation of the dilatation of each rule.

**Dilatation of Baily's Bronze.**—Mr. Chisholm in his account of the construction of the Dominion standards gives the following information (page 9):—

"The coefficient of expansion of Baily's metal for 1° F. at ordinary atmospheric temperatures, was first determined in 1850, by Mr. Sheepshanks, a member of the Standards Commission, to be 0.00000947, and this determination was accepted by the Commission. This coefficient is equivalent to the absolute expansion of 36 inches of bronze for 1° F.=0.00034102 inch. It was obtained from nine comparisons at different temperatures ranging from 36°.06 to 70°.46F. (the actual mean temperature being 56°.55 F.) of the two Standard yards distinguished as Bronze 12 and Brass 2; from which comparisons also the coefficient of expansion of brass for 1° F. was determined to be 0.00000956, equivalent to an absolute expansion of 36 inches of brass for 1° F.=0.00034439 inch.

<sup>\*</sup>The discrepancy between these two figures is accounted for by the fact that Mr. Chisholm used 9.47 x 10<sup>-6</sup> as the thermal dilatation of Baily's bronze and the present result is based on the figure 9.777 x 10<sup>-6</sup>, determined directly for "A" at the Physical Testing Laboratory.

"The results of the several comparisons by Mr. Sheepshanks may be stated as follows:—

Comparisons	Temp	Coefficient		
Comparisons	Extremes	Difference	Mean	of expansion for 1° F.
1) (2) (3) (4) (5)	$\begin{array}{c} 36^{\circ} \cdot 06 - 67^{\circ} \cdot 82 \\ 70^{\circ} \cdot 14 - 45^{\circ} \cdot 85 \\ 41^{\circ} \cdot 95 - 70^{\circ} \cdot 46 \\ 54^{\circ} \cdot 23 - 65^{\circ} \cdot 87 \end{array}$	$31^{\circ} \cdot 76$ $24 \cdot 29$ $28 \cdot 51$ $11 \cdot 64$	51° · 94 58 · 00 56 · 21 60 · 05	0.000009553 9528 9501 9416
lean			56° · 55	0.00000947

"A later determination of the rate of expansion of one of these bronze Standard yards was made in 1865 at Southampton, by Captain Clarke, of the Ordnance Survey Department, and is described by him in the published book, entitled "Comparisons of Standards of Length, London, 1866."

"Capt. Clarke made a series of comparisons between the ordnance bronze Standard yard No. 27 and the Swedish iron yard No. 55, the result being

found as follows:-

Mean result of 50 comparisons from 38°.69 to 99°.16 F., or mean temper	Coefficient of expansion for 1° F.
ature 68° 93	0.0000098870
ature 75°·19	
Mean temperature 72°-06	0.0000098574

"A still more recent determination of the rate of expansion of this alloy has been made in 1874 at Paris by M. Fizeau with his expansion measuring apparatus. Two small pieces of Baily's metal similar to the metal of the Standard yards, were furnished to him by Messrs. Troughton and Simms, and their expansion was observed by M. Fizeau beginning in each case from the lowest temperature of about 10° C. (50° F.) up to higher temperatures varying from 35° C. (95° F.) to 64° C. (147° F.). The results were found to be as follows:—

Number of experiments	Mean temperature	Resulting mean coefficient of expansion for 1° C.	Variation of coefficient for 1° C.	Coefficient of dilatation for 1° C. Mean temper- ature 16° · 6 C.
1st piece 4		0·00001790,325 1779,287	1,350	0.00001765,91
2nd piece 3		1779, 287 1789, 727 1776, 640	1,395	1765, 19
Mean			1,37	0.0000176,55

"Or when reduced to Fahrenheit's scale, a coefficient of expansion at the mean temperature of  $62^{\circ}$  F. of 0.00000980,83, with a variation of 0.76 for  $1^{\circ}$  F. mean temperature.

"It will here be seen that M. Fizeau's determination agrees very nearly with that of Capt. Clarke, and that both assign a higher rate of expansion to Baily's metal than was found by Mr. Sheepshanks. As however each of the three determinations were made with very great care and by experienced observers, but with different portions of the bronze alloy, the conclusion to be drawn is that each Standard bar of this alloy may

have its own rate of expansion, differing more or less from the mean rate, and that the exact rate should be determined for each bar separately in all cases where great accuracy is required. It will also be desirable to determine a mean rate for ordinary temperatures, when the expansion of a sufficient number of bars of this alloy has been authoritatively determined. This mode will also afford the means of determining the mean variation of the rate of expansion of Baily's metal at other ordinary temperatures of the atmospheric air, higher or lower than the temperature fixed upon as normal, say 62° Fahr, or 16° 6°C, at which its mean rate of expansion shall have been ascertained."

In his report on the verification of the yard standards Mr. Chisholm observed that his results indicated that "A" had a coefficient of expansion smaller than that of No. 6. He was not able to make any exhaustive study of the coefficient, however, and suggested that the possibility of changes in the coefficients should be investigated. As the length comparator of the Physical Testing Laboratory is designed for the absolute determinations of thermal dilatations it was accordingly decided to make a study of that of "A".

Method of Determining Thermal Dilatations at the Physical Testing Laboratory.—A rule is placed in the rear tank of the length comparator and is held at room temperature. The variation in the length of this rule will therefore be very slight, especially if, as is usually the case, an invar rule is used. The expansion of rule No. 191, which was employed in determining the dilatation of "A", is only about two parts in one million for a rise in temperature of 1°C. Hence, even apart from the corrections which have to be applied for its slight temperature changes, the rule approximates to a fixed reference length throughout the test.

The rule under observation is placed in the front tank in the ordinary way and a regular comparison made between the lengths of the two rules. The water in the space between the inner and outer front tanks is then cooled by the addition of ice to about 5 degrees below room temperature and when the temperature of the rule has become steady a second comparison is made. Further comparisons are made at temperatures down to  $0^{\circ} \cdot 5C$ , or less. For temperatures above that of the room the circulating water is warmed by the electric heater on the carriage. The usual program is to measure the differences in length between the two rules when the one under observation is at temperatures approximating to  $15^{\circ}$ ,  $10^{\circ}$ ,  $5^{\circ}$ ,  $0^{\circ}$ ,  $5^{\circ}$ ,  $10^{\circ}$ ,  $15^{\circ}$ ,  $20^{\circ}$ ,  $25^{\circ}$ ,  $30^{\circ}$ ,  $25^{\circ}$ ,  $20^{\circ}$ ,  $15^{\circ}$  C., respectively.

During each comparison the microscopes are read just as in the ordinary length observations, but the thermometers in the front tank are read before each set of pointings on the rule being studied, and the water is agitated more frequently.

Results of the Determination of the Thermal Dilatation of the Rule "A".—The thermal dilatation of "A" was determined independently by observers W. J. L. and S. J. M. A typical observation at one temperature is given in Table IV, while Table V shows the difference in length between rule No. 191 (reduced to 16° 667C.) and rule "A" at the temperatures given. The figures for observer S. J. M. are plotted in Fig. 7, in which an amount equal to 8T microns (T being the temperature of "A") is subtracted from each observed difference to increase the vertical scale. The test made by W. J. L. is plotted similarly in Fig. 8.

## TABLE IV

# PHYSICAL TESTING LABORATORY

#### Rule Comparison

Comparison of	T. & S. "A" "	В"
	2 191	38 Date
Observer	W. J. L.	Recorded by G. O. W.
Computed by	G. O. W.	Checked byS. J. M.

#### THERMOMETERS

No	38030	38031	38032	38033	Mean	38036	38037	Mean
Reading	0.47 $0.46$ $0.45$ $0.44$ $0.44$ $0.44$	0.44 $0.43$ $0.43$ $0.43$ $0.42$ $0.42$ $0.42$	0·48 0·48 0·48 0·46 0·48 0·48 0·48	0.45 $0.45$ $0.43$ $0.44$ $0.44$ $0.45$ $0.47$	0·460 0·455 0·447 0·445 0·445 0·447 0·45	16·49 16·50	$16.52 \\ 16.53$	$16.505 \\ 16.515$
Mean	$0.451 \\ -0.041 \\ 0.410$	$\begin{array}{c} 0.427 \\ -0.027 \\ 0.400 \end{array}$	$ \begin{array}{c} 0.477 \\ -0.066 \\ 0.411 \end{array} $	$\begin{array}{c} 0.447 \\ -0.040 \\ 0.407 \end{array}$	$\begin{array}{c} 0.451 \\ -0.044 \\ 0.407 \end{array}$	$\begin{array}{c} 16 \cdot 495 \\ -0 \cdot 018 \\ 16 \cdot 477 \end{array}$	$\begin{array}{c} 16 \cdot 525 \\ -0 \cdot 085 \\ 16 \cdot 440 \end{array}$	16.510 $-0.052$ $16.458$

#### MICROSCOPES

	Left 1	No. 2	Right	No. 1	L —	R
"A"	18·336 18·535 18·712 18·831 18·821 19·168	19·016 19·490 19·620 20·298 20·669	20·772 20·970 21·145 21·274 21·262 21·610	19·082 19·553 19·685 20·367 20·735	$ \begin{array}{r} -2 \cdot 436 \\ -2 \cdot 435 \\ -2 \cdot 433 \\ -2 \cdot 443 \\ -2 \cdot 441 \\ -2 \cdot 442 \end{array} $	-0.066 $-0.063$ $-0.065$ $-0.069$ $-0.066$
Mean. Difference.	18·7338 -1·0	19·8186 0848	21·1722 +1·	19.8844	$-2 \cdot 4383 \\ -2$	-0.0658

Corrected difference.......... -237·26 Microns

TABLE V

DILATATION OF THE DOMINION STANDARD YARD "A"

OBSERVER; S. J. M.

Set	Temperature of "A"	Temperature of 191	Observed difference "A"-191	Correction for 191 to 16°·667 C.	Difference between A at T° and 191 at 16° 667 C.
	$\mathrm{T}^{\circ}$		Microns	Microns	Microns
1 2 3 4 4 5 5 6 6 7 8 9 10 11 11 12 13 14 14	15° 476 C 5 · 998 5 · 618 0 · 763 0 · 695 5 · 238 9 · 454 14 · 529 20 · 074 24 · 196 30 · 114 27 · 187 24 · 018 17 · 171	15°·704 C 16·031 16·048 16·104 16·116 16·148 16·196 16·366 16·496 16·524 16·742 16·742 16·794 16·806	$\begin{array}{c} + & 3 \cdot 91 \\ -149 \cdot 05 \\ -155 \cdot 56 \\ -233 \cdot 51 \\ -233 \cdot 34 \\ -162 \cdot 20 \\ -95 \cdot 22 \\ -13 \cdot 82 \\ +76 \cdot 00 \\ +142 \cdot 62 \\ +238 \cdot 11 \\ +190 \cdot 39 \\ +139 \cdot 17 \\ +29 \cdot 18 \end{array}$	$\begin{array}{c} +1\cdot 72 \\ +1\cdot 14 \\ +1\cdot 11 \\ +1\cdot 01 \\ +0\cdot 99 \\ +0\cdot 93 \\ +0\cdot 84 \\ +0\cdot 54 \\ +0\cdot 31 \\ +0\cdot 26 \\ -0\cdot 13 \\ -0\cdot 49 \\ -0\cdot 23 \\ -0\cdot 25 \end{array}$	$\begin{array}{c} + 2 \cdot 19 \\ -150 \cdot 19 \\ -156 \cdot 67 \\ -234 \cdot 52 \\ -236 \cdot 33 \\ -163 \cdot 13 \\ -96 \cdot 06 \\ -14 \cdot 36 \\ +75 \cdot 69 \\ +142 \cdot 36 \\ +238 \cdot 24 \\ +190 \cdot 88 \\ +139 \cdot 40 \\ +29 \cdot 43 \\ \end{array}$

OBSERVER: W. J. L.

Set	Temperature of "A"	Temperature of 191	Observed difference "A"-191	Correction for 191 to 16° · 667 C.	Difference between A at T° and 191 at 16°·667 C.
	$\mathrm{T}^{\circ}$		Microns	Microns	Microns
1. 2. 3. 4. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15.	14° · 950 C 14 · 359 9 · 502 5 · 361 0 · 523 0 · 407 5 · 423 10 · 248 14 · 921 19 · 961 25 · 492 27 · 935 24 · 754 18 · 739 14 · 075	16° · 271 C 16 · 338 16 · 399 16 · 427 16 · 444 16 · 458 16 · 369 16 · 407 16 · 407 16 · 408 16 · 586 16 · 586 16 · 736 16 · 736 16 · 604	$\begin{array}{c} -3 \cdot 21 \\ -12 \cdot 39 \\ -89 \cdot 73 \\ -156 \cdot 66 \\ -235 \cdot 19 \\ -237 \cdot 25 \\ -157 \cdot 00 \\ -79 \cdot 13 \\ -4 \cdot 46 \\ +78 \cdot 01 \\ +166 \cdot 24 \\ +205 \cdot 37 \\ +153 \cdot 37 \\ +55 \cdot 11 \\ -18 \cdot 95 \end{array}$	$\begin{array}{c} +0.71\\ +0.59\\ +0.48\\ +0.48\\ +0.43\\ +0.40\\ +0.37\\ +0.53\\ +0.47\\ +0.47\\ +0.47\\ +0.36\\ +0.14\\ -0.03\\ -0.12\\ -0.21\\ +0.11\\ \end{array}$	$\begin{array}{c} -3.92 \\ -12.98 \\ -90.21 \\ -157.09 \\ -235.59 \\ -237.62 \\ -157.53 \\ -79.60 \\ -4.93 \\ +77.65 \\ +166.10 \\ +205.40 \\ +153.49 \\ +55.32 \\ -19.06 \\ \end{array}$

The results, within the errors of observation, in each case follow a straight line law, and nothing is to be gained by attempting to compute a quadratic term for the coefficient. The figures obtained were:—

		Observer:	S. J. M.	W. J. L.	Mean
Mean expansion	in microns	per yard per 1° C	$16 \cdot 112$	$16 \cdot 072$	16.092
"	46	metre per 1° C	17.620	17.576	17.598
66	66	metre per 1° F	0.780	0.764	0.777

This result is in close agreement with the figure given by Fizeau as the mean coefficient at  $16^{\circ}$ .667C., and it was used in the reductions of the observations on the length of "A" previously described.

# COMPARISON OF THE LENGTHS OF THE DOMINION STANDARD YARD "A" AND THE PARLIAMENTARY COPY "B"

As has already been stated, Parliamentary copy "B" of the Dominion standard yard is one of the three standard yards received from England in 1874, copy "B" being placed in the custody of the Speaker of the Senate. Unlike the Dominion standard yard "A", copy "B" is only contained in a wooden case which fits inside a wooden box. At some time in its history, probably during or after the fire which destroyed the Houses of Parliament at Ottawa, February, 1916, when all sorts of records and other things were hastily moved from the Senate offices, the outer box was broken open. The seals of the Senate and of the Standards Department, Board of Trade, England, can still be deciphered, however, and the rule itself does not appear to have been damaged in any way. Viewed under the microscopes of the length comparator the graduation lines on "B" are more regular than those on "A".

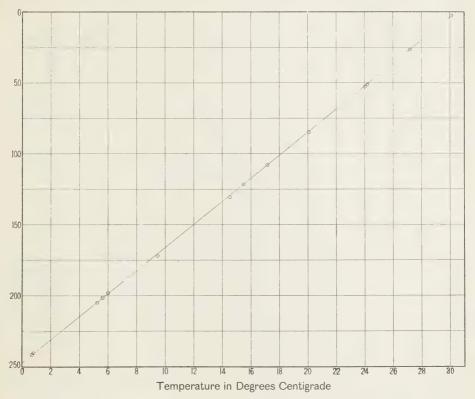


Fig. 7

Tests.—"B" was first compared with "A" directly, and then compared in a closed series made up of rules "B", No. 224 and No. 306. Only one observer (W. J. L.) participated in these tests.

In the comparison of "A" and "B" the rules were compared in the usual eight positions, the procedure being exactly similar to that already explained in connection with the study of "A".

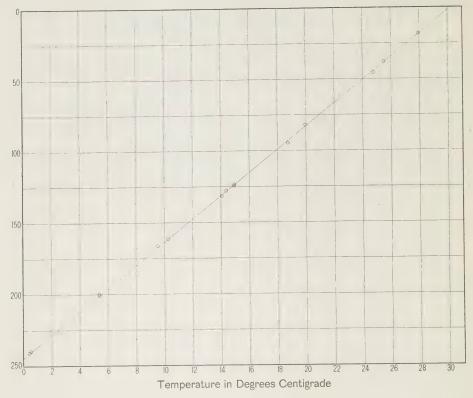


Fig. 8

Table VI gives the results of this test. It was assumed that "A" and "B" had the same thermal dilatation, which, unless the true coefficients varied considerably (an unlikely condition), would introduce no appreciable error owing to the tests being conducted near the actual temperature of standardization.

Table VIII shows the result of the closed series observations. From this comparison "B" appears to be 0.000820 inches shorter than Rule 306 and 0.000528 inches longer than rule 224, giving the following lengths for "B" at  $16^{\circ}.667$ C. in terms of the Imperial Yard:—

From rule No. 224 "B" = 35.999670 inches.

From rule No. 306 "B" = 35.999671 inches.

Using the value of the verification of "A" already determined, we get from the direct comparison of "A" and "B"—

"B" = 35.999666 inches at  $16^{\circ}.667$ C.,

which agrees very well with the value determined directly from rules 224 and 306.

Taking the mean of the two determinations as being the present length of "B" at  $16^{\circ}.667$ C., i.e. 35.999668 inches, "B" has apparently shortened by 0.000278 inches since 1874. Assuming that the coefficient of thermal expansion of "B" is equivalent to that determined directly for "A", the present length of "B" at  $16^{\circ}.756$ C. (62.16F.) becomes 35.999724 inches.

TABLE VI

Direct Comparison of the Dominion Standard Yard "A" and the Parliamentary Copy "B" of the Yard

Position	Temperature	Length of "A"  -Length of "B"
1	16° · 304 C 16 · 318 16 · 326 16 · 338 16 · 351 16 · 384 16 · 385 16 · 391	Microns 7 · 60 7 · 77 7 · 37 8 · 14 7 · 34 6 · 75 8 · 04 7 · 95 7 · 62

#### TABLE VII

Comparison between the Parliamentary Copy "B" of the Yard and the Physical Testing Laboratory Rules No. 224 and 306, in a closed series

All differences are reduced to 16° · 667 C.

(a)

Position	Temperature	Length of Rule No. 306 -Length of Rul No. 224
		Microns
1	$\begin{array}{c} 16^{\circ} \cdot 535 \text{ C} \\ 16 \cdot 535 \\ 16 \cdot 538 \\ 16 \cdot 534 \\ 16 \cdot 544 \\ 16 \cdot 562 \\ 16 \cdot 541 \\ 16 \cdot 551 \end{array}$	$34 \cdot 68$ $34 \cdot 53$ $34 \cdot 57$ $35 \cdot 02$ $34 \cdot 45$ $34 \cdot 28$ $34 \cdot 40$ $35 \cdot 09$
eandjusted mean	16°·542 C	34 · 63 34 · 22

(b)

Position .	Temperature	Length of Rule No. 306 -Length of Rule "B"
		Microns
1	16° · 688 C 16 · 682 16 · 679 16 · 678 16 · 669 16 · 674 16 · 670	20·36 20·68 20·31 20·44 20·28 20·43 20·34 20·49
Aean	16°·677 C	20·42 20·83

#### TABLE VII-Concluded

(e)

Position	Temperature	Length of Rule "B"  -Length of Rule No. 224
		Microns
1	16° · 755 C 16 · 742 16 · 750 16 · 745 16 · 731 16 · 742 16 · 738 16 · 728	$\begin{array}{c} 13 \cdot 02 \\ 12 \cdot 90 \\ 12 \cdot 80 \\ 12 \cdot 76 \\ 12 \cdot 65 \\ 13 \cdot 04 \\ 13 \cdot 68 \\ 13 \cdot 10 \\ \end{array}$
eandjusted mean	16° · 741 C	$12 \cdot 99 \\ 13 \cdot 40$

#### EXAMINATION OF THE BRONZE STANDARD METRE

A single metre standard rule was sent to Canada with the other Dominion standards in 1874. In form and material this rule is similar to the three standard yards. It is marked:—

"Mr. Baily's metal" "Standard Metre" "Troughton and Simms, London".

The Mètre des Archives.—In 1874 the Mètre des Archives was the standard of length for the metric system. The Mètre is a bar of pure platinum one metre in length, 25 millimetres wide and 4 millimetres thick made in 1795. It represents the unit at a temperature of 0°·C., by the distance between the centres of its end faces. With a desire, like that of the framers of the Act of Parliament of 1824 establishing the Imperial yard, for defining the unit in terms of a constant of nature, it was originally intended to make the metre a one ten-millionth part of a quadrant of a meridian of longitude measured on the earth's surface, but somewhat similar considerations of uncertainty in the determination of the value of the constant resulted in the metre being referred solely to the length of the platinum standard.

The International Metre.—The Mètre des Archives was replaced as standard in 1889 by the Standard International prototype metre. This latter is a line standard, composed of platinum-iridium alloy and made with the X or Tresca cross-section. The International metre is to-day the standard of length for the metric system throughout the world, and the national metric standards, distributed to all those countries participating in the establishment of the International metric standards, are prototypes of the metre which have been carefully studied at the International Bureau of Weights and Measures, Sèvres, France.

The length defined by the International standard metre at 0°C. is declared to be identical with that of the Mètre des Archives, to within the errors incident to the comparison between an end standard and a line standard.

Position of the Metre in Canada.—The Weights and Measures Act of 1873 legalized the metric system, and in Schedule A set forth the relation between the metric measures and Imperial measures. At the same time provision was made permitting the acquisition of metric standards. In the Act of 1873 the legal relation was

1 Metre=3.281833 feet

The bronze metre standard was obtained from England in 1874 with the other standards and in the Act of 1879 is referred to as "The Dominion Standard

for determining the length of the Metre . . . ."
In 1914 a short Act (4-5 George V, Chap. 4) was passed which established the International metre as the basic unit of length for the metric system in Canada. This Act also gave revised figures for the relation between the Imperial and Metric units, the new value for the metre being

# 1 Metre=3.280843 feet,

which is the legal relation observed in England at the present time, although recent comparisons between the yard and the metre have resulted in a slightly different figure being obtained.

The Act of 1914 was to come into force when copies of the International metric standards, duly certified by the International Bureau of Weights and Measures, had been obtained and authorized by Order in Council. As these standards are not yet to hand, it would appear that the old bronze rule is still the legal standard metre for Canada and the old relation between the yard and the metre holds good.\*

The Canadian Bronze Standard Metre.—The bronze metre already referred to was verified by Mr. Chisholm in 1874, when he compared it with the bronze subdivided metre SS of the Standards Department.

The length of this standard had been compared about 1870 with the platinum line standard of the Royal Society and was said to be 0.00313 millimetres longer than the Mètre des Archives at 0°C. although it is not stated how the length of the Royal Society's standard was obtained.

The following test results are given by Mr. Chisholm, the Canadian metre being denoted by BM:-

Number	Date ·	Temper- ature of compared bars	Results of Comparisons	Bar next Observer
1 2 3 4 5	1874, Nov. 11, 12.30 P.M	54°·6 F 37·5 52·6 52·1 51·3 37·5	BM = SS -0.01138 mm -0.00632 -0.01240 -0.01174 -0.01213 -0.00678	BM SS SS SS BM BM

Whence, at mean t 52° 6 BM = SS - 0.01191 mmBM = SS - 0.00655 mmWhence, at mean t 37°.5

From these results Mr. Chisholm concluded that the Canadian Metre has a smaller coefficient of thermal expansion than SS, and using the two results just given, obtained by extrapolation:—

BM at  $32^{\circ}$  F=1 Metre -0.00147 millimetres.

A series of further experiments indicated that the Canadian Metre had a coefficient of expansion equal to  $0.9205 \times 10^{-5}$  per 1° F., for a mean temperature of 56° · 9 F.

Results of the Examination of the Canadian Bronze Standard Metre at the Physical Testing Laboratory.—The bronze metre rule was compared in the length comparator directly with Rule No. 224, No. 224 being at about 16° · 7 C., the temperature at which it was verified at Sèvres, and the bronze

<sup>\*</sup> The situation has been changed by the Revised Statutes 1927, Chap. 212, in which is incorporated parts of the 1914 Act, reference to the old bronze standard being dropped.

rule at a temperature of about  $0^{\circ} \cdot 5$  C. A second test was made with the metre in a closed series with rules Nos. 224 and 306 at a temperature of about  $16^{\circ} \cdot 5$  C. In the first test the two rules were compared in four relative positions (it being impossible to interchange them in the tanks) and in the second the regular

eight position program was followed. The observer was W. J. L.

The lengths of the one-metre intervals of the Laboratory rules were determined at Sèvres in 1924 when the two rules were compared in a closed series with prototype No. 26 of the International metre. The two tests made at the National Physical Laboratory indicate that the yard intervals have remained practically constant since 1924, and hence the same assumption may be made with confidence respecting the metre intervals. The lengths determined at Sèvres are the mean of two independent tests, one made by Mons. L. Maudet (of the International Bureau) and the other by the writer. The figures are:—

Length of one metre interval on Rule No. 224 at  $0^{\circ}$ C=1 Metre + 8.54 microns.

Length of one metre interval on Rule No. 306 at  $0^{\circ}$ C=1 Metre + 17.53 microns.

The temperature of test was close to  $16^{\circ} \cdot 7$  C.

 $0.654~\mathrm{C}$ 

Mean.....

The following is the result of the first test on the Canadian Bronze Metre:—

Set	Temperature of 224	Temperature of Bronze Metre	Difference Bronze Metre — 224 (at T <sub>1</sub> °) (at T°)	Correction for 224 to 0° C.	Difference Bronze Metre – 224 (at T <sub>1</sub> °) (at 0° C.)
	T°	T <sub>1</sub> °	Microns	Microns	Microns
1 2 3	16·720 C 16·741 16·766 16·791	0.683 C 0.641 0.622 0.670	$-118 \cdot 79$ $-119 \cdot 13$ $-119 \cdot 99$ $-120 \cdot 04$	$\begin{array}{r} -125 \cdot 25 \\ -125 \cdot 41 \\ -125 \cdot 59 \\ -125 \cdot 78 \end{array}$	$+6.46 \\ +6.28 \\ +5.60 \\ +5.74$

TABLE VIII

i.e. assuming a linear thermal dilatation law for the bronze metre, this rule at  $0^{\circ} \cdot 654$ C. is  $6 \cdot 02$  microns longer than rule No. 224 at  $0^{\circ}$ C.

+6.02

In order to compute an approximate value of the coefficient of expansion of the bronze metre, before reducing the observations of the second test, the comparison between the metre and rule No. 224 were taken in conjunction with the first test. Table IX below gives the differences between the bronze metre at about 16°·5C. and rule No. 224 at 0°C, as computed from the observations in the second test.

TABLE IX

Set	Temperature	Difference Bronze Metre-224	Correction for 224 to 0° C.	Difference Bronze Metre — 224 (at T°) (at 0° C.)
	$\mathrm{T}^{\circ}$	Microns	Microns	Microns
1	16 · 520 C 16 · 511 16 · 510 16 · 511 16 · 511 16 · 546 16 · 548 16 · 546	$\begin{array}{c} +159 \cdot 28 \\ +159 \cdot 06 \\ +159 \cdot 38 \\ +159 \cdot 38 \\ +158 \cdot 59 \\ +160 \cdot 80 \\ +160 \cdot 77 \\ +160 \cdot 65 \end{array}$	$\begin{array}{c} -123.76 \\ -123.69 \\ -123.68 \\ -123.69 \\ -123.95 \\ -123.95 \\ -123.95 \\ -123.95 \end{array}$	+283·04 +282·75 +283·22 +283·07 +282·28 +284·75 +284·74 +284·60
Mean	16·525 C			+283.56

i.e. with the same assumption regarding the dilatation being linear, the bronze metre at 16°.525°C. is 283.56 microns longer than rule No. 224 at 0°°C., which in combination with the test shown in table VIII gives an expansion of 277.54 microns for a rise in temperature of 15.871 degrees. The mean coefficient of expansion derived from this result is:—

 $17 \cdot 49 \times 10^{-6} \text{ per } 1^{\circ}\text{C.}$ or  $9 \cdot 71 \times 10^{-6} \text{ per } 1^{\circ}\text{F.}$ 

#### TABLE X

Summary of the Comparisons of the One-metre Intervals on Rules Nos. 224 and 306 with the Canadian Bronze Metre Standard (BM)

All differences are reduced to 0° C.

G-1	306-	-BM	224—BM		306-224	
Set	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test
1	14 · 14 13 · 25 13 · 62 13 · 47 14 · 45 14 · 13 13 · 83 14 · 15	16° · 270 C 16 · 301 16 · 282 16 · 296 16 · 320 16 · 343 16 · 361 16 · 370	5.89 $6.03$ $5.54$ $5.71$ $6.50$ $4.65$ $4.67$ $4.79$	16° · 520 C 16 · 511 16 · 510 16 · 511 16 · 511 16 · 546 16 · 548 16 · 546	9.42 $8.83$ $9.10$ $8.74$ $8.89$ $9.11$ $9.21$ $9.02$	16°·619 C 16·605 16·600 16·596 16·592 16·614 16·608 16·609
Mean	13.88	16°⋅318 C	5.47	16° ⋅ 525 C	9.04	16° ⋅ 605 C
Adjusted mean.	14·09 Microns		5·26 Microns		8.83 Microns	

Table X gives the full results of the test of the bronze metre rule in a closed series. In computing these results the figure just arrived at is used to reduce the differences to a temperature of 0°C. The length of the bronze metre becomes as follows, in terms of the International metre:—

From rule No. 224 Bronze metre at 
$$0^{\circ}$$
 C. = 1 Metre  $+ 3.28$  Microns " " " = 1 Metre  $+ 3.44$  Microns Mean = 1 Metre  $+ 3.36$  Microns

These figures rest on the validity of the coefficient of expansion derived from the direct comparisons with rule No. 224. If at any future date a more complete study should be made of this coefficient of expansion, sufficient data is given to allow the length of the bronze metre derived from the comparison in closed series to be recomputed.

## EXAMINATION OF THE BRONZE YARD No. 16

Bronze yard No. 16 was sent to what was then the Colony of Canada when the bars which had been used by the British Restoration of Standards Committee were distributed throughout the world. This yard is a duplicate of the bar chosen for the Imperial standard yard (No. 19 of the series studied by the Committee) and is marked in precisely the same way. It is inscribed:—

Tin 2½ Mr. Baily's Metal No. 16 Standard Yard at 61°.94 Faht. Cast in 1845. Troughton and Simms, London Zinc 1

The defining lines are clear and regular, appearing under the microscope to be much superior to those on the Dominion standard "A". At the left end (with regard to the inscription) there are two dark patches which obscure part

of the centre line, but the middle portion of the line defining the length of the bar is in good condition. There are some slight scratches on the gold plugs which, however, do not affect the use of the bar as a length standard, and otherwise No. 16 is in an excellent state of preservation.

Having regard to the age of the bar, its close relationship to the Imperial standard yard, the reputation Mr. Sheepshanks had for the diligence and accuracy with which he made his comparisons, a test of the present length of

No. 16 is of great interest.

Test.—No. 16 was compared with the standards Nos. 224 and 306 in a closed series in October, 1927, by S. J. M. The mean temperature of the comparisons was around 17°C. and it was assumed that No. 16 had the same coefficient of thermal expansion as "A". The results of the test are given in Table XI. Using these figures we get for the present length of No. 16:

Hence No. 16, when compared with the Imperial standard yard, is within 0.00008 inch of its difference from the standard as determined about 1855.

#### TABLE XI

Summary of Results of the Observations in the Test of Bronze Yard No. 16  ${\rm All~differences~reduced~to~16^\circ\cdot667~C.}$ 

OBSERVER: S. J. M.

Set	306 - 224		306-16		224-16	
	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns	Temperature of test	Difference in Microns
1	17°·166 C 17·166 17·166 17·169 17·181 17·189 17·195 17·205	$34 \cdot 27$ $34 \cdot 51$ $34 \cdot 86$ $34 \cdot 41$ $34 \cdot 32$ $34 \cdot 23$ $34 \cdot 23$ $34 \cdot 93$	16° · 774 C 16 · 776 16 · 805 16 · 818 16 · 839 16 · 851 17 · 099 17 · 106	13 · 56 13 · 58 13 · 79 14 · 11 13 · 91 13 · 48 13 · 44 13 · 82	17°·105 C 17·114 17·136 17·142 17·152 17·162 17·161 17·172	$\begin{array}{c} 19 \cdot 93 \\ 20 \cdot 30 \\ 19 \cdot 88 \\ 20 \cdot 05 \\ 19 \cdot 71 \\ 20 \cdot 36 \\ 20 \cdot 25 \\ 20 \cdot 25 \end{array}$
Mean	17°⋅180 C	34.32	16° · 884 C	13.71	17°·143 C	20.09
Adjusted mean.	34.15		13.88		20.26	

